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ON THE CONSTRUCTION OF THE SECOND AVENUE LINE OF THE METROPOLITAN ELEVATED RAILWAY OF NEW YORK.

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READ APRIL 20TH, 1881.

In presenting a paper upon the Construction of the Second Avenue Line of the Metropolitan Elevated Railway in New York, of which Mr. Wm. F. Shunk is Chief Engineer, the writer, who has been the Division Engineer in charge of the work, feels impelled to apologize to the Members of the Society for its length and the recitation of numerous details, not easily avoided, however, in a comprehensive treatment of the subject. It has been written in the midst of the busy daily duties of professional labor.

DESCRIPTION OF THE ROUTE.

The line departs from the joint structure of the Metropolitan Elevated Railway and the New York Elevated Railroad at Chatham square, by curves of 230 and 250 feet radii for the up and downtown tracks

respectively. Expressed metrically, with radii of 70.10 and 76.198 metres. The uptown track has 55° 30′ 45″ and the downtown track 36° 10′ 40″ curvature; thence through Division street by tangent, excepting the uptown track, which has a reverse curve of 8° 18′ total curvature and radius of 1 910 feet (582.157 m.), to form a parallel track; thence into Allen street by curves of 210 feet (64.007 m.), radii, with curvature of 63° 03′; thence through Allen street by tangent to Houston street; thence by curves of 1 910 feet (582.157 m.), radii and curvature of 7° 41′ into First avenue; thence by tangent through First avenue to Twenty-third street; thence with a curvature of 90° and radii respectively of 136 feet and 110 feet (41.452 and 33.527 m.), on the up and downtown tracks; thence by tangent through Twenty-third street to Second avenue, and into Second avenue by curves similar to those at First avenue and Twenty-third street, excepting that those of the up and downtown tracks of the former correspond respectively to those of the down and uptown tracks of the latter; thence by tangent through Second avenue to One Hundred and Twenty-ninth street or the Harlem river. (See Plate VIII.)

TABLE No. 1.

TABLE OF LENGTHS OF TANGENTS AND CURVES.

LOCATION.	EAST TRACK.		WEST TRACK.	
	Feet.	Metres.	Feet	Metres.
Chatham square Curves	222.80	67.908	157 80	48.907
Division street Tangent	1 004.60	306.196	991.10	302.082
Allen street Curves	231.12	70.444	231.12	70.444
Allen street Tangent	3 206.67	977.375	3 209 61	978.271
Houston street Curves	256.10	78.058	256.10	78.058
First avenue Tangent	5 448.38	1 660.636	5 438.50	1 657.625
First avenue Curves	219.90	67.024	172.79	52.665
Twenty-third street Tangent	503.00	153.311	503.00	153.311
Second avenue Curves	172 79	52.665	219.90	67.024
Second avenue Tangent	27 942.00	8 516.566	27 939.00	8 515.651
Total lineal feet and metres	39 207.36	11 950.183	39 118.92	11 923.228
Equals miles and kilometres	7.42 miles	11.950 kil. m.	7.409 miles	11.923 kil. m.

The curves are but little more than two per cent. of the length of the whole line.

The successive steps in the construction of an Elevated Railway are not wholly different from those which occur in building on the surface, as will be seen by the following natural order to which we conformed, viz. :

1. Preliminary survey.
2. Location of "layout."
3. Construction of foundations.
4. Setting cast-iron bases for columns.
5. Erection of iron structure.
6. Track structure and track laying.
7. Painting.

PRELIMINARY SURVEY.

In order that no mistakes should be made in projecting a layout, or in the succeeding work of construction, this survey should be made, as it was, with the greatest care. The obstacles to the attainment of such a result in the crowded business streets of a city like New York are only known to Engineers whose patience and good nature have similarly been taxed quite up to the limit of elasticity.

It may be surprising to some to learn that no errors greater than one inch per one thousand feet (.0254 m. per 304.795 m.) of horizontal measurement, and five-thousandths of a foot per one foot (.0016 m. per .305 m.), in elevation were allowed.

The first, and a very important step in the survey, was the adoption of a convenient base line that could be used until the final completion of the railway. It was located on the easterly sidewalks far enough from the curb line to escape any possible interference from the excavations for piers. No small difficulty was experienced in locating these base lines, especially on the avenues, where they are from one to five miles long, the requirements being a straight line parallel to the axis of the street or avenue. There is no angle in the First avenue base line, but to overcome several irregularities in the Second avenue line, and to prevent encroachment with our columns on the horse-car tracks, a few slight angles of a few seconds each were permitted on the base line; these, where it was possible to do so, were located on summits of grades, and cannot be detected with the eye.

After permanently establishing the base lines, the longitudinal dist-

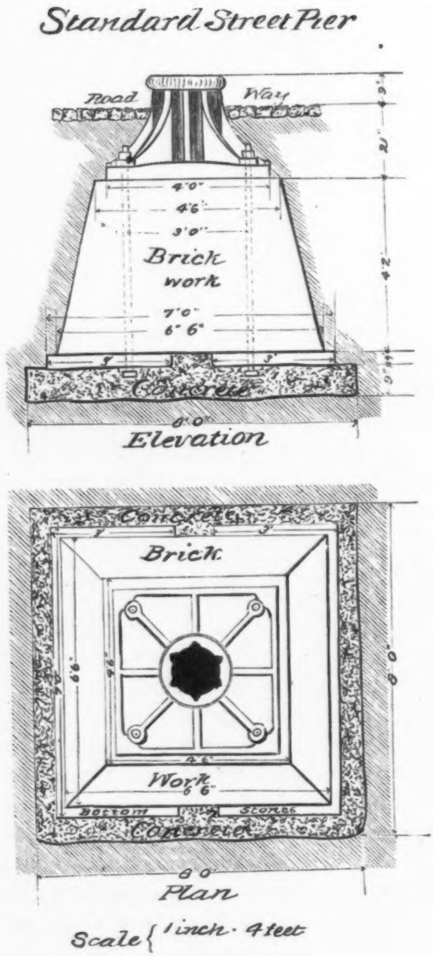
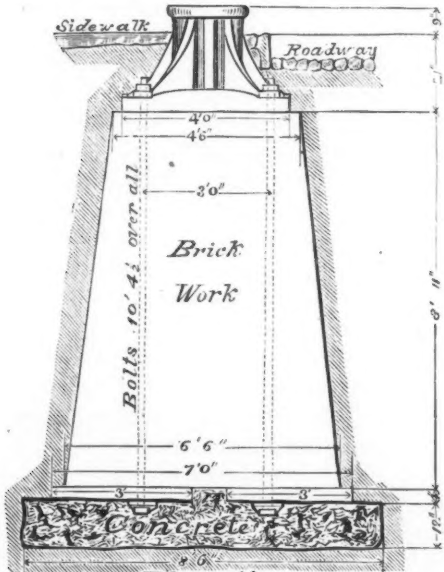
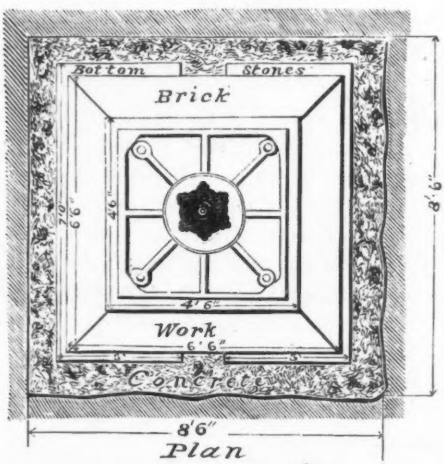

Standard Street Pier
Road
Way
Brick
work
Concrete
Elevation
Concrete
Brick
Work
Bottom
Stones
Concrete
Plan
Scale { 1 inch · 4 feet

PLATE IX.

Standard Sidewalk Pier



Elevation



Plan

Scale { 1 inch = 4 feet; 1 mm = 48 mm }

PLATE X.

ance was measured with wooden rods 25 feet (7.619 m.) in length, the measurement being guided by a transit man. These rods were made by Young of Philadelphia, and pronounced by him to be accurate by the United States standard. Several auxiliary test standard lengths were first laid off at convenient points, where the rods were tested every morning for the day's measurement. Full stations were thus located and permanently marked every one hundred feet (30.479 m.) and sub-stations every twenty-five feet (7.619 m.) Following the measuring party was another small party, whose duty it was to locate all obstacles with steel tape from some of the full or sub-stations. It was found of the utmost importance in the subsequent projection of a layout that all such prominent features as sewer basins, fire hydrants, man-holes, lamp posts, telegraph poles, sewer, gas, and water mains, curb lines, house lines, vaults, chimneys, horse-car tracks, and large, important and public buildings, should be accurately measured and mapped. A location for the columns was then decided upon, and spotted upon the map for construction or verification. The spacing and arrangement of the columns on the street or sidewalk to support the structure is called the "layout." After the columns had been marked off at their proper sites, elevations thereat were taken for the purpose of establishing grade and determining the length of the columns.

For the purposes of construction, bench marks were permanently established every two blocks, or about five hundred feet (152.396 m.) apart. The profile of the Road is shown in Plate XXII.

CONSTRUCTION OF FOUNDATIONS.

The requirements imposed by the Board of Commissioners of Rapid Transit in regard to foundations were :

1. The foundations, where necessary, shall be increased in area, so that in no case shall a weight greater than two thousand pounds to the square foot come upon any base (9765. kgs. per sq. m.)

2. Good hydraulic cement shall be used in the masonry, and Portland cement of the first quality shall be used in case the foundations are constructed of concrete. If bricks are used, they shall be hard burnt, and of the best quality.

No foundations were constructed of concrete ; all the other requirements were rigidly adhered to. Their classification was as follows :

1. Standard street piers.

PLATE XI
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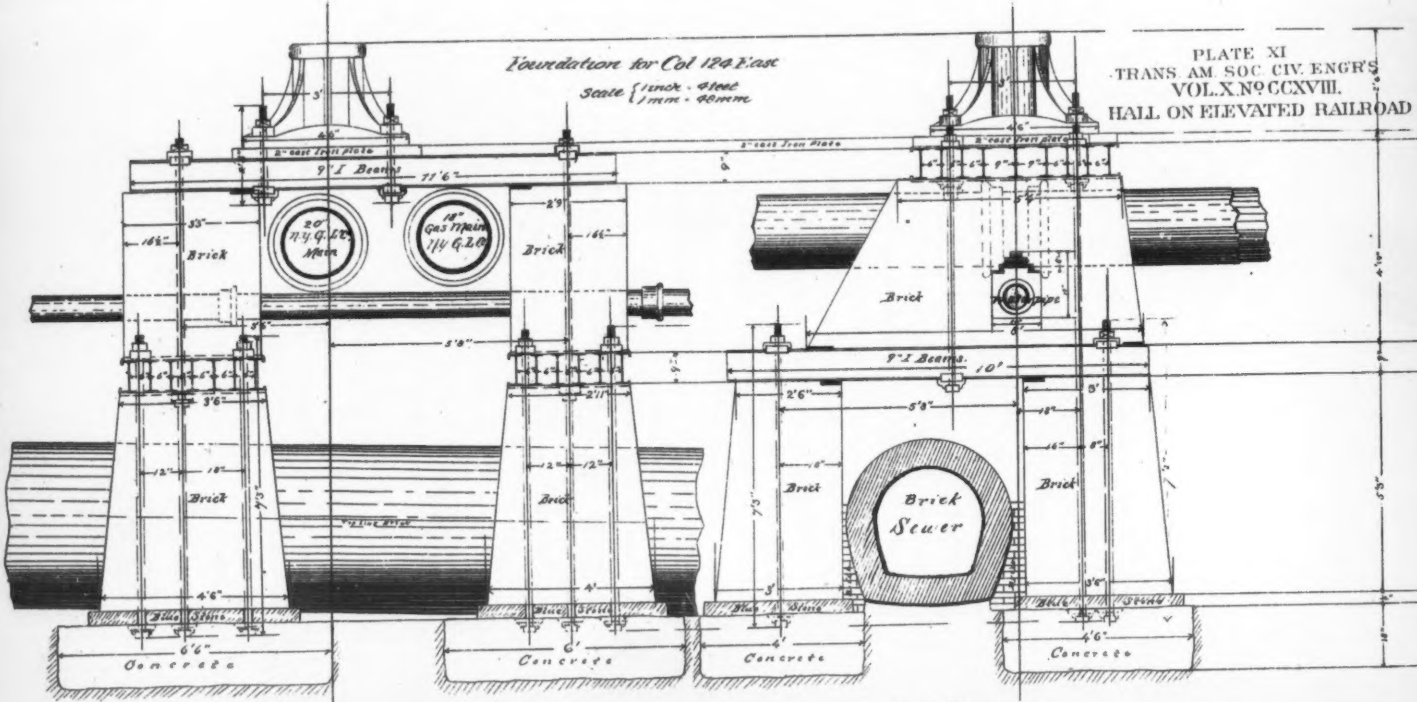

South Elevation

East Elevation

2. Standard sidewalk piers.

3. Special street piers.

4. Special sidewalk piers.

The standard street pier was built in the driveway on firm ground where no obstacles were met. The pits were opened nine feet (2.743 m.) square, and excavated to a depth of five feet six inches (1.676 m.) below the bottom of the base casting, which usually made the excavation about seven feet (2.133 m.) below the street surface. The bottom of the pit was then examined with a sounding rod, and if found satisfactory, was leveled and where deemed expedient, compacted with rammers. The sides a foot (.305 m.) high from the bottom, were then shaped and smoothed to enclose a bed of concrete eight feet (2.438 m.) square and nine inches (.2287 m.) thick. The concrete was composed of broken stone, not exceeding two and one-half inches (.0635 m.) in the largest dimension, *four parts*; clean, sharp sand, *two parts*; American hydraulic cement, *one part*. The cement and sand were first mixed with a minimum amount of water, after which the broken stone, being well wet, was added, the whole mixture being then thoroughly incorporated on a tight board platform, was deposited in one layer in the pit, and rammed till the soft material flushed to the surface; before setting, pockets were scooped in the surface of the concrete, which were partially filled with thin hydraulic mortar subsequently to receive the bolt washers and heads. On top of the concrete were laid two stones, averaging five inches (.127 m.) thickness and three feet by seven feet (.914 m. by 2.13 m.) laid so as to offer a base of seven feet (2.13 m.) square. These large stones served the purpose of anchorage for the bolts as well as support for the brick pier. Plate IX. gives plan and elevation of standard street pier; Plate X, standard sidewalk pier.

Standard Sidewalk Piers. Pits for the piers were excavated 12 feet (3.66 m.) deep. The concrete footing was 8 feet 6 inches (2.59 m.) square, and 12 inches (.305 m.) thick, deposited in two layers. The height, from bottom of concrete to bottom of socket, in base casting, was 10 feet 6 inches (3.2 m.)

The height from bottom of foundation stones to top of brickwork was 9 feet 3 inches (2.82 m.); in all other respects the piers corresponded to the standard street piers previously described.

Special Street Piers were those which from natural or artificial causes

could not be built to the standard plan. The reasons for departing from the regular form of construction were foundations in quicksand, muck, made ground, or other soft bottom, adjacent or underlying water and gas mains, sewers, with their appendages of man holes, basins, culverts, &c.

Special Sidewalk Piers were those required when vaults or other subterranean obstacles were encountered.

The total number of piers built to date is 2 014—of which 1 496 are special. Of the latter number one hundred and ninety-eight rest on piles; the total number of piles driven was 3 310; the average length driven was 20 feet (6.1 m.) They were spruce, straight, straight-grained and smooth, free from large knots, divested of bark, not less than twelve inches (.305 m.) at the butt after sawing off, and eight inches (.201 m.) at the small end. They were driven to hard bottom, or until the set did not exceed one fourth of an inch (.0063 m.) under the blow of a ram weighing twelve hundred pounds (544.3 Kg.) falling eight feet (2.44 m.) The piles were hooped for driving, and sawed off horizontally one foot (.305 m.) below mean high tide level, their heads then being surrounded with concrete, the usual form of capping and flooring was used.

Elevations of four street piers and an isometrical drawing of one are given in Plates XI., XII., XIII. and XIV., as fair samples of the prominent obstructions encountered on First and Second avenues. The frequency of the obstacles met with is made apparent by the fact that $74\frac{3}{10}$ per cent. of the piers are special.

The materials for concrete and hydraulic mortar were compounded by actual measure, and all cement was subject to inspection and test before use. The specifications for cement required it to be capable of resisting a tensile strain of fifty pounds to the square inch (3.52 Kg. per square cm.) after thirty minutes exposure in air and twenty-four hours immersion in water. The results of some tests made by the inspector of sand and cement are given in Exhibit 1, and show results from cement obtained from five different manufacturers. Much good sand was obtained from the excavations at different points along the work. The remainder was delivered on scows from "Cow Bay," Long Island, or "The Kill," on Staten Island. The latter, although much more difficult to obtain, was by far the better sand; both kinds, however, were required to be "bank sand."

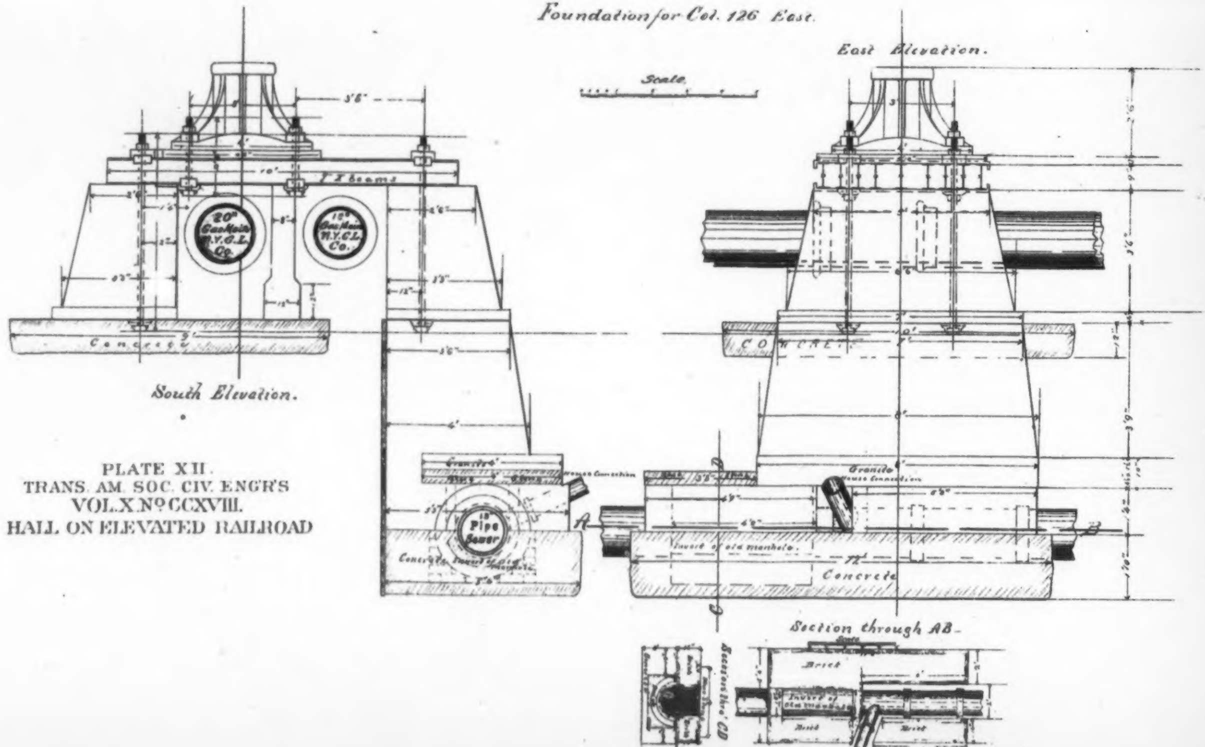

PLATE XII.
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[illegible] Second Avenue Extension M. E. R.

EXHIBITS OF TESTS OF ROSENDALE CEMENT, USED ON SECOND AVENUE EXTENSION M. E. R.

Brand of Cement.	Proportion of Mixture: Time of Setting: Breakage per Square Inch.												Temperature.		Fineness.
	Neat.			Sand 1; Cement 1.			Sand 1½: Cement 1.			Sand 2: Cement 1.					
	Time.	Breakage.		Time.	Breakage.		Time.	Breakage.		Time.	Breakage.				
		Lbs. per sq. in.	Kg. per sq c m.		Lbs. per sq in.	Kg. per sq c.m.		Lbs. per sq. in.	Kg. per sq. c. m		Lbs. per sq. in.	Kg. per sq. c. m.	Air.	Water	
F. O. Norton	24 hrs	44	3.09	24 hrs.	40	2.81	24 hrs.	40	2.81	48 hrs.	39	2.74	Avg. tem. 73° F.	68° F.	96:100
"	"	50	3.52												
"	"	42	2.95												
"	"	58	4.08	"	58	4.08	"	48	3 38	"	40	2.81		"	
"	"	50	3.52												
"	"	65	4.57	48 hrs.	60	4.22	48 hrs.	50	3.52	"	45	3.17		"	
"	7 days.	80	5.63	7 days.	60	4.22	7 days.	40	2.81	7 days.	40	2.81			
"	2 mths.	360	25.32	2 mths.	220	15.47	2 mths.	189	13.29	2 mths.	174	12.24			
"	5 "	560	39.38												
"	6 "	530	37.27												
Newark & Rosendale	24 hrs.	44	3.09										74° F.	70° F.	95:100
"	"	46	3.24	24 hrs.	40	2.81	7 days.	49	3.45						
"	48 hrs.	42	2.95												
"	24 hrs.	38	2.67	"	36	2.53	"	52	3.66	2 mths	114	8.02			
"	"	42	2.95	"	34	2.39									
"	"	45	3.17	"	32	2.25	1 mth.	175	12.31						
"	2 mths.	325	22.86	......	......	......	......	......	......	5 mths.	275	19.34			
New York Cement Company	24 hrs	42	2.95	24 hrs.	31	2.18	24 hrs.	36	2.53	......	......	......	74°	70°	91:100
" "	48 hrs.	42	2.95												
" "	7 days.	62	4.36	......	......	......	......	......	......	......	......	......	72°	68°	
" "	24 hrs.	48	3.38												
" "	"	48	3.38	48 hrs.	38	2.67									
" "	"	50	3.52	......	......	......	24 hrs.	28	1.97	24 hrs.	30	2.11	71°	59°	
Hoffman & Lawrenceville	24 hrs.	48	3.38	......	......	......	......	......	......	24 hrs.	32	2.25	74°	70°	92:100
"	"	52	3.66												
"	"	42	2.95												
"	7 days.	68	4.78	......	......	......	......	......	......	7 days.	38	2.67	70°	67°	
Rocklock	24 hrs.	39	2.74												94:100
"	"	58	4.08												
"	"	62	4.36	......	......	......	24 hrs.	38	2.67	7 days.	87	6.12	78°	70°	
"	"	47	3.31	......	......	......	7 days.	94	6.61	......	......	......	"	"	

These mixtures and most of the tests (except the long time tests) were made during May, June, July and August, 1879, from the cement as received from the mills. A much larger number of tests, however, were made. From accepted brands only a portion have been selected for this tabulation, which are probably sufficient to show the strength of the Rosendale cement used on short and long time tests for tensile strength.

Since writing the bulk of this paper the writer had occasion to build some twenty piers to connect the Metropolitan Elevated Railway on Eighth avenue with the Harlem River Bridge. As the cost of materials and labor have largely increased since the construction of the Second avenue line, an epitome of this work is shown in Appendix I.

It may be of interest to note some experiments made by the writer to ascertain the amount of shrinkage in the materials combined to make the hydraulic mortar used. The measurements of the sand, cement and water were carefully made in a box containing exactly one cubic foot (28.3 litres). It was found that in the mixture, including the water that was used, there was a total shrinkage of 37 and $38\frac{6}{10}$ per cent., respectively, for two different tests. Omitting the water in the deduction, it was found there was a shrinkage of $22\frac{1}{2}$ and 25 per cent. in the dry materials, after being mixed into mortar for use.

The following table shows the proportions used:

TABLE No. 2.

QUANTITIES IN CUBIC FEET.

Cement.	Sand.	Water.	Mixture.	Mortar Made.	Remarks.
4	8	$2\frac{3}{4}$	$14\frac{3}{4}$	$9\frac{3}{10}$	Experiment (1)
3	6	2	11	$6\frac{3}{4}$	" (2

QUANTITIES IN LITRES.

113.3	226.5	77.8	417.6	263.4	Experiment (1)
85.	169.9	56.6	311.5	191.1	" (2)

From these results it would appear that for any known quantity of cement mortar required, 25 per cent. of the amount should be added to the dry materials for shrinkage, the materials being used in the foregoing proportions.

PLATE XIII.
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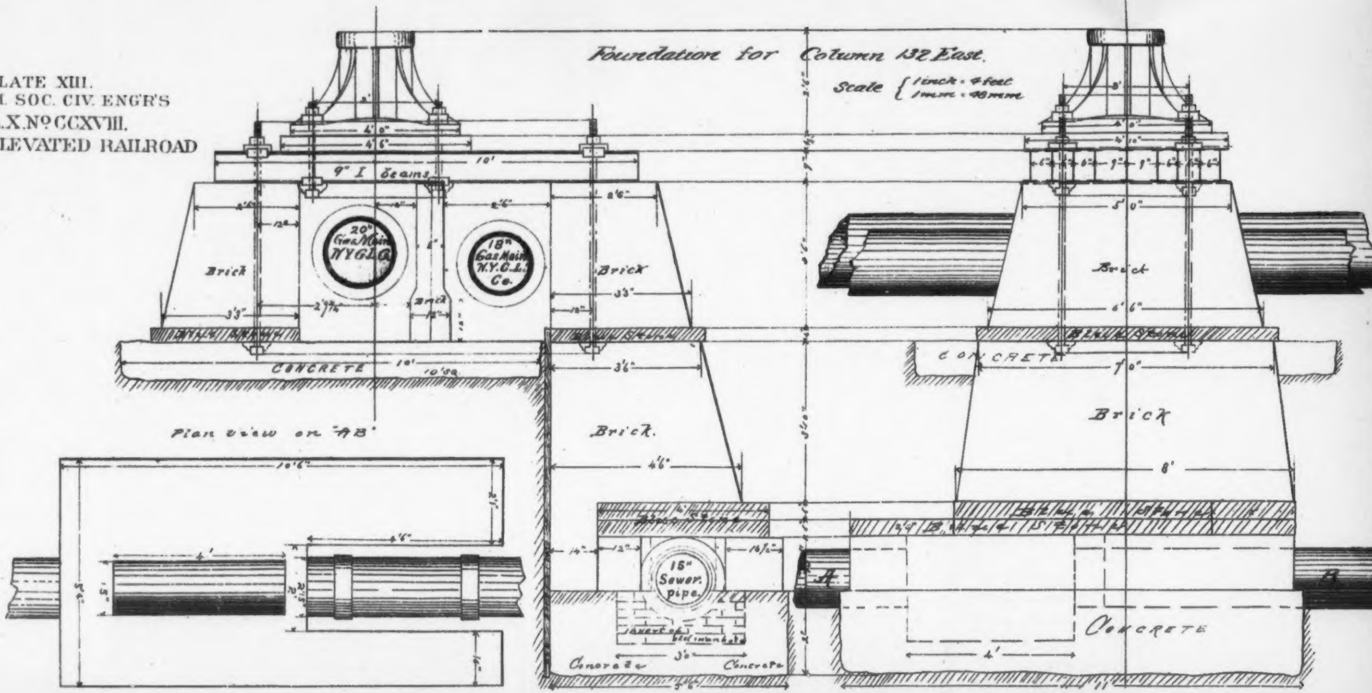

The total number of bricks used in foundations, including the station piers built to date (Feb. 1, 1881), is 14 841 198, or an average of 7 369 per pier. The Haverstraw brick, as laid in these foundations, numbered by actual count from 19 to 21 per cubic foot (0.671 brick per litre), the variation being probably due to the thickness of joint.

Cast Iron Base Castings.

There were six patterns of base castings, used and designated as follows: A, AD, AE, B, BD, and BE. The calibre of the first three patterns is $13\frac{1}{2}''$ (.343 m.) for Phœnix columns, whose cross section area ranges from 36 to 39 square inches (232.3 to 251.6 sq. c. m.)

The calibre of the remaining three patterns is $13\frac{3}{4}''$ (.35 m.) for the same style of column, having a cross section area of 42 to 45 square inches (271 to 290 sq. c. m.) As the different patterns of the two calibres are essentially the same, only differing one fourth of an inch (.0063 m.) in diameter, sketches of three patterns only are shown in Plate XV.

The base castings were delivered by the company on the curb line opposite the piers where they were to be set. The proper elevation, together with the lateral and longitudinal adjustment of the base, was given by the engineer in charge, who also directed how the work should be done. The restoration of the street surface was made in all cases acceptable to the city street authorities.

The following tables show the actual cost of material and labor for setting the base castings by days' work, where the pier had been built and street restored before the base casting was set:

TABLE 3.

FOR A AND B BASE CASTINGS.

			TIME.	COST.
Time and cost of		uncovering pier—2 men at 15c. per hour	$1\frac{1}{8}$	.35
"	"	moving base from sidewalk to pier—5 men at 15c. per hour..............	$\frac{1}{2}$	.40
"	"	erecting derrick and setting base casting—5 men at 15c. per hour.............	$\frac{4}{5}$	.60
"	"	repaving 25 square feet - 2 men at 25c..	1	.50
"	"	washing, tarring and bricking—2 men at 25c...........................	$\frac{2}{3}$	.35
"	"	refilling—2 men at 15c................	$\frac{1}{2}$	.15
"	"	preparing cement mortar—1 man at 20c.	$\frac{1}{2}$	.10

Foreman and night watchman........................	.50	2.95
Amount of cement used—¼ bbl. at $1....	.25	
" sand " —¼ bbl. at $1.25 per cubic yard..	.05	
Number of bricks used–32 at $10 per M....	.32	
Amount of refuse carted away—½ cubic yard............	.38	
" sand for paving 2½ cubic feet.................	.11	
" coal tar, cement wash, &c................ ...	.10	
Oil for lamps, and incidentals..........................	.05	1.26
Total cost....................		$4.21

TABLE NO. 4.

FOR AD, AE, BD, AND BE BASE CASTINGS.

	TIME.	COST.	
Time and cost of uncovering pier—2 men at 15c. per hour	1⅔	.50	
" " moving base from sidewalk to pier—5 men	¾	.60	
" " erecting derrick and setting base casting—5 men..........................	1¼	.94	
" " repaving 40 square feet—2 men at 25c. per hour..............................	1½	.75	
" " washing, tarring and bricking—2 men at 25c. per hour......................	1	.50	
" " refilling—2 men at 15c. per hour........	¾	.23	
" " preparing cement—1 man at 20c. per hour	¾	.15	
Foreman and night watchman..........................		.63	4.30
Amount of cement used—½ bbl. at $1..................		.50	
" sand " —½ bbl............................		.10	
Number of bricks used—60 at $10 per M...............		.60	
Amount of refuse carted away—¾ cubic yard...........		.56	
" sand for paving 4 cubic feet.................		.18	
" coal tar, cement wash, &c..................		.15	
Oil for lamps, and incidentals..........................		.05	2.14
Total cost..................			$6.44

Where the base setting followed the construction of the brick piers, it was found that thirty-five per cent. of the above cost was saved.

The base castings north of Eighty-seventh street were set the year

PLATE XIV.
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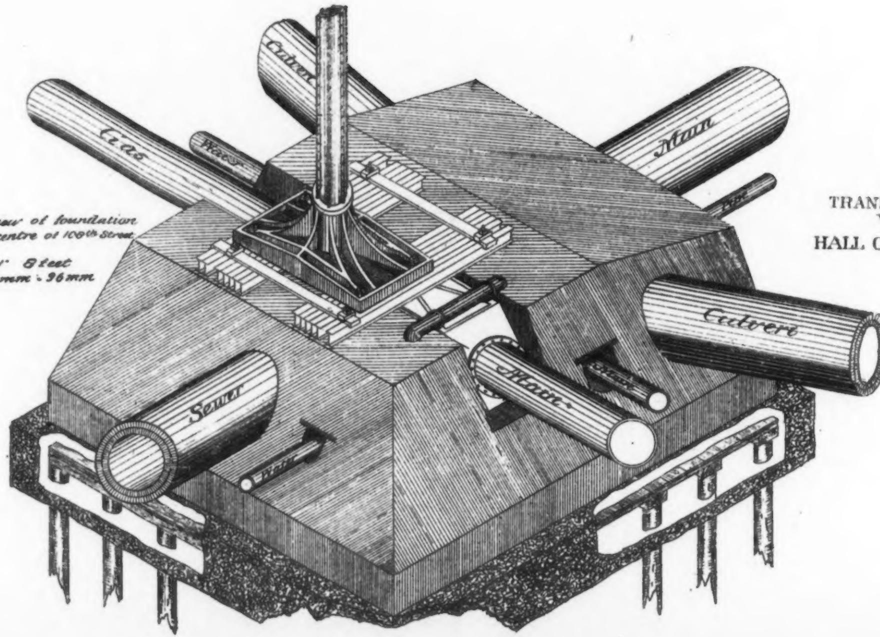

Isometrical view of foundation for Col. 708 W. centre of 100th Street

Scale 1" = 8 feet
" 1 mm = 96 mm

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following the pier construction, and done by contract at the following prices:

For A and B base castings, including all labor and materials $3.75

For AD, AE, BD and BE base castings " " 5.25

A saving of about eleven per cent. on the former, and about eighteen per cent. on the latter style of bases.

The average number set per day by days' work or by the company was ten, while the average number set per day by contractor was fifteen. It might be well to state, in this connection, that the contractor gave his undivided, personal attention to the work.

The following table shows the average weight of the different kinds of bases used, number of each pattern, and total weight of all in tons of 2 240 pounds and 1 000 kilograms :

TABLE No. 5.

WEIGHTS OF CAST IRON BASES.

No. Used.	Pattern.	Average Weight.		Total Weight.		
		Lbs.	Kgs.	Tons of 2 240 lbs.		Tons of 1 000 kilograms.
				T.	Lbs	
1 463	A	3 204	1453.32	2 092	1 336	2126.21
49	A D	5 533	2509.75	121	0 420	122.98
50	A E	6 026	2733.37	134	0 840	136.67
409	B	3 205	1453.78	585	0 854	594.60
24	B D	5 510	2499.32	58	1 840	59.98
5	B E	6 048	2743.35	13	1 300	13.72
2 000				$3005\frac{94}{100}$ tons.		3054.16

The specifications for cast-iron column bases required them to be made of the best No. 2 Lehigh pig metal, or its equivalent in quality, three tons to one ton of machinery scrap.

The mixture was tested by taking a bar therefrom one inch (.0254 m.) square, and one foot (.305 m.) between supports, which was required to sustain at its middle point one gross ton (1016 kgs.), without fracture, the casting to be perfect in shape, the bolt-holes spaced exactly and cored square to the base. The cores for the barrels to be made

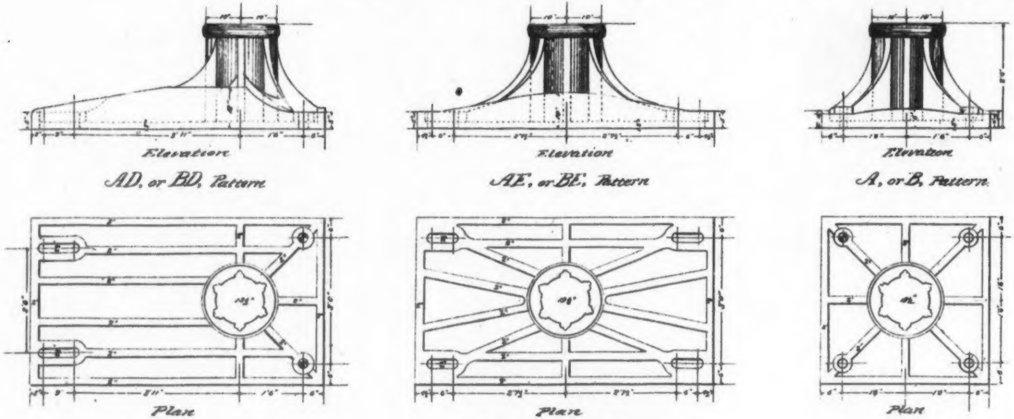
without draught, and blackened to get a smooth casting ; great care was necessary in adjusting the barrels, and the spherical bearing at the bottom of the socket was cast chilled. The casting was then permitted to cool slowly, remaining in the sand not less than twenty-four hours. The bases described, by their peculiar form of construction, were difficult to cast. The distribution of the metal in the barrel, bottom of socket, ribs and flanges, while well calculated to resist the incumbent strains upon them, necessitates great care in founding. The iron, in cooling, shrinks along the ribs, while the expansion, owing to the large amount of metal in the centre of the base, still continues at the bottom of the socket.

These reactionary forces sometimes caused a bursting of the base. Other reasons, also, combined to produce similar results, such as the use of too hard iron, pouring while too hot, cooling the casting too quickly, or moulding in damp sand, the latter tending to ruin the core by expansion of air or steam, or both, and thereby rendering the bottom of the socket too thin for use. The bases were subjected to a rigid inspection by striking with a sledge, measuring the various parts and scrutinizing with a critic's eye before leaving the foundry. The final act of setting the base in position was also a good test of its perfect construction. Not a few were rejected in the last act. The bases were set to a given elevation—an elevation of top of base being previously determined for the whole line, chiefly dependent upon the grade of the streets. A bed of hydraulic mortar, from one-half to three-fourths of an inch (.0127 to .0191 m.), was given them that they might always be brought to a true elevation and to furnish a perfectly even bearing when screwed down to the brick pier. The ends of the bolts, together with the nuts, were then covered with a thick coat of coal-tar, and the whole base given a wash of pure cement mortar where the bases occur in the driveway. They were further protected by cast-iron bell-shaped fenders, which also prevent the wheel hubs of passing vehicles from striking the iron columns. The intervening space between the shell of the fender and the base casting was filled with cement mortar. Plate XVI. represents a fender in position, the base and column being shown by dotted lines.

Erection of Iron Structure.

The iron structure is composed of Warren longitudinal girders of the deck system, whose upper chords rest upon the top chords of transverse

Patterns of Bases used on 2nd Ave. Line



Scale { 1 inch = 4 feet. 1 mm = 48 mm

girders of a similar character, supported by six segment wrought-iron columns of the Phœnix pattern. The transverse girders have a double triangular system, whereas the longitudinals are of the plain triangular type. The longitudinal girders are set in pairs under each track, five feet (1.524 m.) between centres. They are four feet (1.219 m.) deep from out to out of chord angles.

The bottom chords of transverse girders are flush with the under surface of longitudinals, and made rigid therewith by plates and bolts. The transverse girders through Division and Allen streets, First avenue and Twenty-third street, are sized and fitted for the support of three tracks, and through Second avenue for four tracks.

Transverse girders weigh about 200 pounds per lineal foot (297.4 kg. per m.)

Longitudinal girders weigh about 130 pounds per lineal foot (193.5 kg. per m.)

Bracing, &c. weigh about 8 pounds per lineal foot (11.9 kg. per m.)

"A" calibre columns weigh about 350 pounds per yard (173.7 kg. per m.)

"B" calibre columns weigh about 420 pounds per yard (208.4 kg. per m.)

The first step in the erection of the iron work was dropping the columns into their appropriate base castings. This was easily accomplished by means of a derrick wagon, shown in Plate XVII, by which seven men and a team of horses were able to set in place an average of thirty-nine columns not exceeding 20 feet (6.096 m.) in length; or, ten columns not exceeding 50 feet (15.240 m.) in length per diem. The space between the barrel of the base casting and the columns was then caulked with oakum to keep out the water prior to using the cast-iron cement, and the columns filled with cement mortar in proportion of two parts sand to one of cement.

The columns were held temporarily in place by inserting iron wedges inside the rim of the base casting. The girders were raised by a traveler on the structure. The power used was a 15-horse power engine, boiler shell 3′ × 6′ (.914 m. × 1.828 m.). The force needed was twelve men besides the engine man. With the power and force named the contractor raised an average of sixty-six girders per diem of ten hours, equaling 200 tons (203.22 tons kg.), using one-fourth ton (253.11 kg.) of coal. (See Plate XVIII.)

Fender

Used on 2nd Ave.

Scale 1" = 1 foot
" 1mm = 12mm

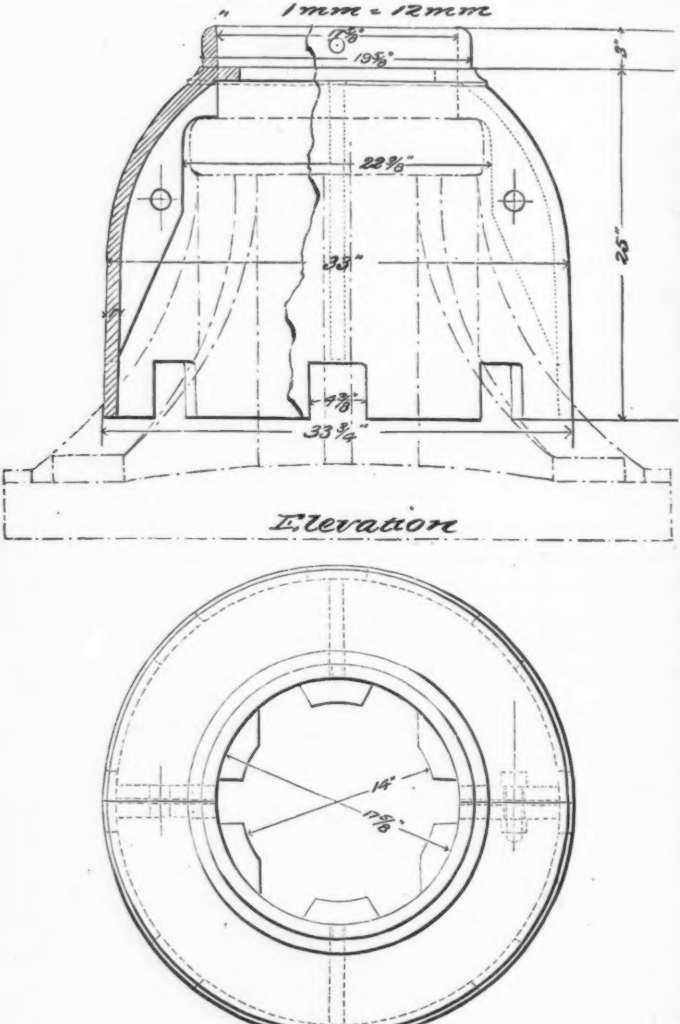


PLATE XVI.

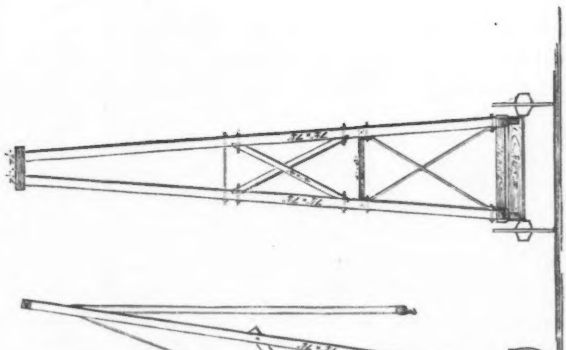


PLATE XVII.

The iron girders all being in place, the lateral bracing was then adjusted and riveted, and the columns very carefully plumbed with heavy iron plumb-bobs, and in many cases of long columns with a transit instrument. After being brought to a vertical position they were thus held by jack-screws and braces until the cast-iron cement had been substituted for the oakum.

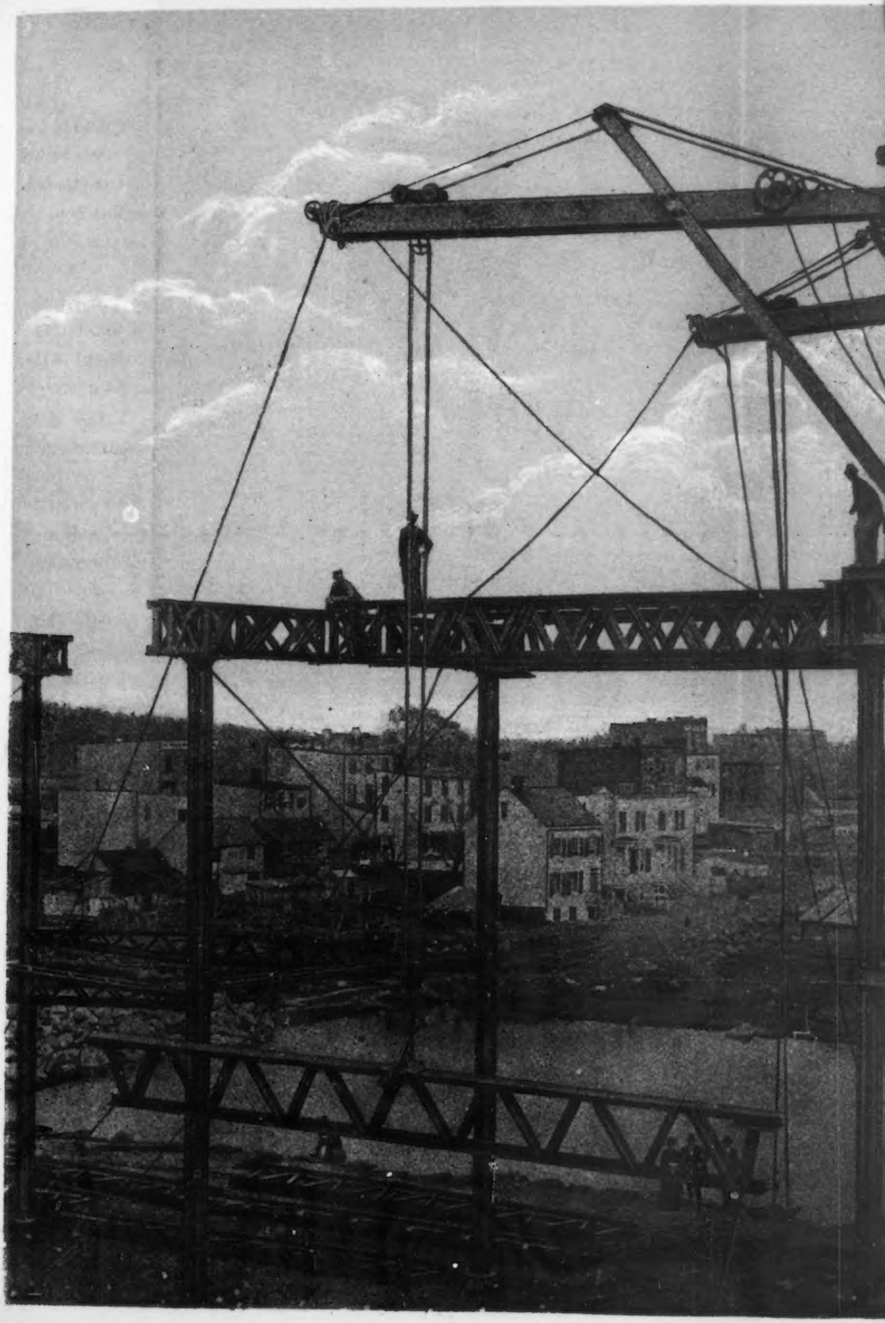
The cast-iron cement used consisted of the following proportions: One ounce (28.3 g.) of sal-ammoniac to one gallon (3.79 l.) of water. The cast-iron turnings or borings were mixed with a minimum amount of the solution, and left standing about half an hour before using, to "warm up." The mixture was then dropped in layers of $\frac{1}{8}''$ to $\frac{1}{2}''$ (.0032 m. to .0127 m.), not to exceed the latter, and very thoroughly rammed with iron rammers driven by vigorous blows of a hammer. For a "B calibre" base about 130 pounds (58.97 kg.) of borings were required to perfectly cement the column and its base coating. For a base of "A calibre" about 100 pounds (45.36 kg.) were needed. It took about forty-eight hours to complete all evaporation, whereupon the cement became as hard as the iron itself. Much care was needed in preparing this cement. Unless sufficient water was used the cement would heat and be spoiled. The addition of sulphur one part to eight parts of sal-ammoniac hastens the process of hardening, but if time is allowed it sets more firmly without the sulphur. Hence, the use of sulphur was abandoned. It was also found absolutely necessary that the iron borings should be perfectly clean and free from rust. As an illustration of the efficiency of this cast-iron cement it may be interesting to know that when occasion offered a column was removed from a cast-iron base after it had been thus cemented, whereupon it was found stronger than the base itself, the latter breaking first under powerful blows of a sledge.

I.—Track Structure and Track Laying.

(1.) The gauge of the track on tangents is 4′ 8½″ (1.435 m.). On curves of 600 feet (182.9 m.) or less it is 4′ 9″ (1.448 m.).

(2.) The track materials consist of—

(*a*.) Southern yellow pine cross-ties 6″ × 6″ × 8′ (.1524 m. × .1524 m. × 2.438 m.), except on curves, crossings and turnouts, where



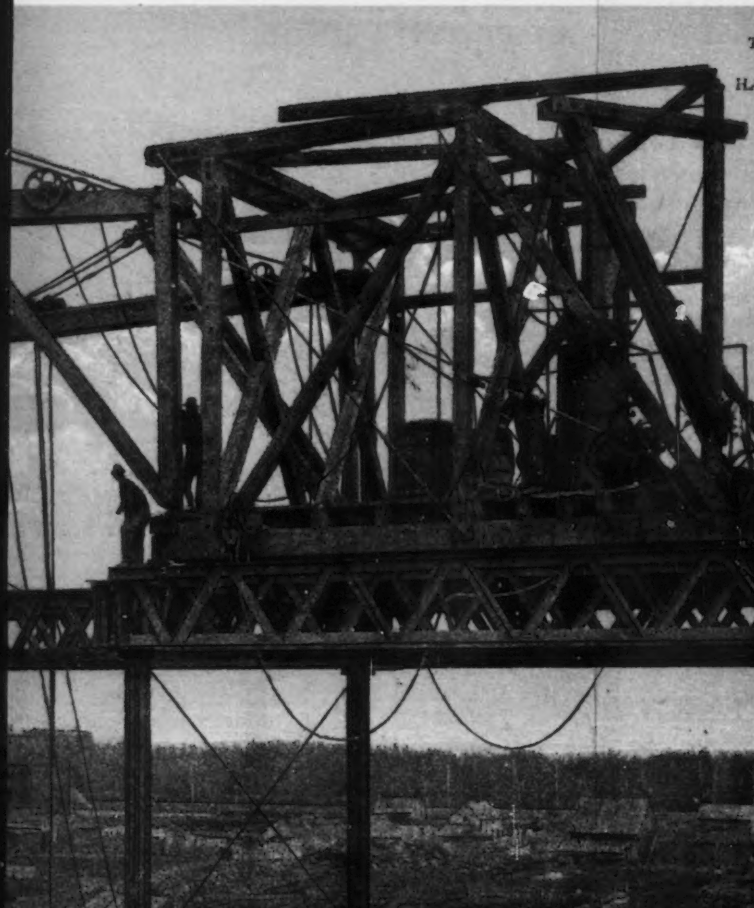



PLATE XVIII
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greater lengths and larger sections were frequently used. Every third cross-tie on each track of the main line is a lookout tie 12′ (3.657 m.) long.

(*b.*) Southern yellow pine guard timbers 6″ × 8″ × 30′ (.1524 m. × .2032 m. × 9.14 m) long and upwards for straight lines, and white oak guard timbers of like lengths 5″ × 8″ (.127m. × .2032 m.) section for sharp curves and their approaches.

(*c.*) White oak bevel and distance blocks, cap stringers and shims for sharp curves.

(*d.*) Steel rails weighing 63 pounds per yard (31.26 kg. per m.), and usually 30 feet (9.14 m.) long.

(*e.*) Fish plates, screw bolts, lag screws, clips, angle bar and strap-iron guards, blunt bolts, spikes and nails. All materials were delivered along the line, in the street, by the company, the contractor thereon receipting for the same and assuming all further responsibility. The accepted bids for track work were those of Messrs. D. R. Kelly, and Quinn & Flynn, all of New York City. The following table shows the amounts and respective prices credited to each :

TABLE No. 6.

CONTRACT PRICES PAID FOR TRACK WORK, AND AMOUNT DONE BY EACH.

Names.	In Lineal Feet.										Remarks.
	Tangents.		Curves.		3 Plank Walk.		Switches.		Frogs.		
	Price.	Length.	Price.	Length.	Price.	Length.	Price.	Number.	Price.	Number.	
D. R. Kelly.....	43c.	50764.4	80c.	1628	....	......	$3 00	35	$4 00	41	
Quinn & Flynn.	35c.	37840.	....	......	3c.	31982	12 00	23	3 00	26	Turnouts and crossings all estimated as tangents. Switches to include furniture, and in working order.
	In Metres										
D. R. Kelly.....	$1.41	15481	$2 62½	496.2	....	......	$3 00	35	$4 00	41	
Quinn & Flynn.	1.48	11533	......	......	8½c.	9748	12 00	23	3 00	26	

II.—Straight Lines.

The cross-ties were spaced 18″ (.4572 m.) from centre to centre, excepting at rail joints, where shoulder-ties were used, spaces 6″ (.1524 m.) in the clear, from the joint tie. They were laid at right angles to the centre line of the track, with the outer ends in range. Wrought-iron clips and lagscrews were used to fasten them to the flanges of the longitudinal girders, two clips and two lagscrews to each cross-tie alternately inside and outside of the girders. The top of the cross-tie over each transverse girder was always true grade; whenever camber showed itself in the longitudinal girders it was taken out by adzing the under surface of the cross-ties. No shimming was allowed in any case. The rails are those known to the trade as "Erie Pattern," made of Bessemer steel with $\frac{35}{100}$ of one per cent. of carbon.

They were furnished by the Albany Rensselaer Iron and Steel Company, and met satisfactorily all requirements of the company's inspector. A test bar $\frac{3}{4}$″ (.019 m.) square, and one foot (.305 m.) long was taken from each charge and required to bend 80° without breaking, under a sudden pressure, on bearings of 8″ (.2032 m.) asunder. Two bars out of three to stand this test in order that the charge might be rated first class. If two out of three broke the charge was rated second class, and rejected. An official test record was kept of all charges, and open to the company's inspection. A template for section was furnished by the company. It was found that although the manufacturers were very successful, generally, in meeting the requirements of the specifications, yet it was quite common to discover a slight "kink" near the end of the rail; this could not well be avoided with the straightening machine then in use at the mills.

They are placed to break joints in each track, the ends of them, on one side, laid as nearly as possible opposite the middle of those on the other side, and the joint made exactly over the middle of the tie, each rail was then spiked with two spikes in each cross-tie. The joints were left open $\frac{1}{4}$″ (.0064 m.) when the temperature was below Fahrenheit zero: a scant $\frac{1}{4}$″ (.0064 m.) between zero and 32° : $\frac{3}{16}$″ (.0048 m.) between 32° and 50° : $\frac{1}{8}$″ (.0032 m.) between 50° and 80° ; and $\frac{1}{16}$″ (.0016 m.) above 80°. Only metal shims were used in regulating the spaces.

The guard timbers were laid both sides of the steel rails, and exactly 10″ (.254 m.) apart, the inner one being $3\frac{1}{2}$″ (.0889 m.) in the clear

from the gauge side of the track rail when complete, with strap iron. They were framed at the ends with a dovetail halving splice 9" (.2286 m.) long, tapering one inch (.0254 m.) The splices in all four guard timbers were made to break joints; screw-bolts were used to fasten the timbers at the joints. Each guard timber was, as a general rule, bolted to alternate cross-ties, the inside and outside timbers being bolted in pairs respectively to the same tie, they were secured to the girder flanges by clips above the nuts of the screw bolts, the heads and washers of which were set down into a cup at top, bored with a 2" (.0508 m.) auger, deep enough to sink the head slightly below the top surface of the guard timber and permit the screw end of the bolt to project about ⅛" (.0032 m.) below the nut after screwing up for upsetting. The auger-hole cup was filled completely with a paste of Portland cement, and smoothed off a little crowning to shed water, prior to painting. All the bolts used in track-work, excepting fish-plate bolts, were greased with tallow before insertion, and all meeting surfaces of track timbers and other fixtures were given a thick coat of white lead and oil paint before closing up.

III.—Curves.

The general provisions concerning straight lines governed track-work on curves as far as they were applicable. The yellow pine ties were usually 10 feet (3.048 m.) long. To give the proper elevation of the outer rail, white oak bevel caps were used for the support of rails and guard timbers. The outer rail was elevated 3" (.0762 m.) calculated for a speed of ten miles per hour only; the elevation being gained in a distance of about 80 feet (24.4 m.) on the tangent to the curve. The track-rails were curved by templets furnished by the engineer, true to line; no springing or sledging of rails was allowed. On curves with radii of 600 feet (183 m.) and upwards each rail is held by three spikes per tie —two outside and one inside. On curves of less than 600 feet (183 m.) radius, a steel guard rail was spiked down inside of the inner track rail; it was found necessary to plane off the flanges of this rail ¾" (.0191 m.) to permit the insertion of the screw bolts, or lagscrews; then steel guard rails were further supported and held upright in place against lateral shocks by white oak distance blocks, moulded to fit their sides, and extended nearly to the opposite guard timber, where they were "set up" by means of a wedge of oak; they were then bolted fast through the

bevel blocks and cross-ties. On top of these distance blocks were bolted through cross-tie a white oak curved cap 3¾" × 10" (.0953 m. × .254 m.) section, partly covering the head of the steel guard rail; the inner face being 3½" (.0889 m.) from the gauge side of the inner track rail. This was guarded also on the upper edge facing the track rail by a 5" × 3" (.127 m. × .0762 m.) angle bar secured by blunt bolts. These angles were in about 14 feet (4.27 m.) lengths shaped to the curve with half check joints. The three remaining ranges of guard timbers for each track were composed of two white oak timbers, each 5" × 8" (.127 m. × .2032 m.) section clamped together sideways by bolts and lag screws, at intervals of about 1 foot (.305 m.). They were then fastened to the cross-ties as on straight lines. Both ranges of guard timbers for the outside rails on curves were faced with strap iron, the timbers being spaced 6½" (.165 m.) from the gauge side of the track rail instead of 6" (.1524 m.) as on tangents. The ends of the cross-ties were then sawed off true to the curve. Figs. 1 and 2, Plate XIX, represent sections of the track for tangents and sharp curves. Owing to the fact of longitudinal girders on sharp curves being chords of the arc, and the consequent means of fastening the cross-ties varied, the different sections are more or less dissimilar.

It may be of interest to know the organization which was maintained during the construction of this novel and somewhat exceptional class of track-work. As the work was necessarily continuous, or was deemed best to be so, it was found that the most economical daily force employed should not exceed seventy men, distributed over a space of about half a mile, as follows:

15 carpenters,
10 skilled laborers to assist the same on guard timbers,
10 men laying steel rails,
10 " clipping cross-ties,
10 " spacing, marking and adzing cross-ties,
10 unskilled laborers for derrick, distributing materials, etc,
2 horses with drivers,
3 foremen.

The clippers were usually kept about 500 feet (152.4 m.) ahead of the spikers, and the spikers 750 feet (228.6 m.) ahead of the carpenters on the guard timbers. Horse power was found to be cheaper and of more practical utility than steam in hoisting track material. The cross-ties

PLATE XIX
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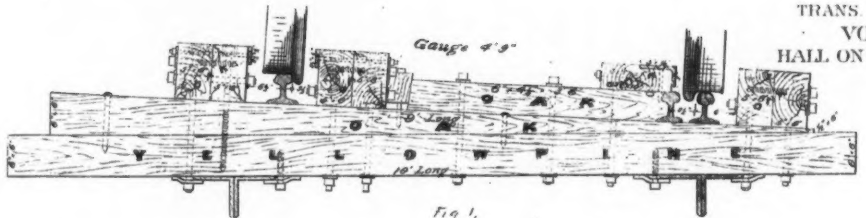


Fig 1.

Scale 1" - 2 feet
1mm - 24mm

Fig 2

were first hoisted, distributed and spaced; then marked for camber by means of T sights, adzed and clipped. Then the steel rails were in turn distributed, lined up and spiked; then the guard timbers were distributed, ends jointed, gauged and bolted down; the inside guard being put in place and finished with strap iron before the outside one was laid. A space of about 250 feet (76 m.) intervened between the gangs employed on the two ranges of guard timbers. Everything working smoothly, the above force accomplished about one block or 260 feet (79.25 m.) of double track *per diem* of ten hours, on tangent work.

The following statement gives the actual cost to contractors Quinn and Flynn of laying complete one thousand lineal feet (304.8 m) of straight single track :

Hoisting and distributing materials	$40 00
Laying cross-ties	65 00
Hoisting steel rails	30 00
" guard timbers	100 00
Strap ironing guard timbers	20 00
Incidentals, loss of time, repairing tools, etc.	25 00
Superintendence	20 00
	$300 00
Amount paid contractor as per prices bid	350 00
	$50 00 Profit.

or $16\frac{2}{3}$ per cent.

The amount of materials required for the construction of one thousand feet (304.8 m) of single track, is as follows :

250 cross-ties, 6″×6″×12′ (.1524 m. × .1524 m. ×3.658 m.).
500 " 6″×6″×8′ (.1524 m. × .1524 m. ×2.438 m.).
3 000 wrought iron clips $5\frac{1}{2}$″×$2\frac{1}{2}$″ ×$\frac{1}{2}$″ (.1397m. × .0635 m. × .0127 m.)
1 500 lag screws, 6″×$\frac{3}{4}$″ (.1524 m. × .0191 m.).
67 steel rails, 30′ (9.144 m.) long.
67 fish-plates, 20″×$2\frac{1}{2}$″×$\frac{3}{4}$″ (.5082 m. ×.0635 m. × .0191 m.).
268 " " bolts, 4″×$\frac{3}{4}$″ (.1016 m. × .0191 m.) with nuts and washers.
3 000 spikes.
7 000 lineal feet guard timber, 6″×8″ (.1524 m. × .2032 m.).
1 500 guard rail bolts, nuts and washers, $14\frac{1}{2}$″×$\frac{3}{4}$″ (.3664 m. × .0191 m.).
150 lag screws, 12″×$\frac{3}{4}$″ (.305 m. × .0191 m.).

2 000 lineal feet strap-iron, $2\frac{1}{2}'' \times \frac{1}{2}''$ (.063 m. × .0127 m.) section.
300 strap iron bolts, $6\frac{1}{2} \times \frac{1}{2}''$ (.1651 m. × .0127 m.) with nuts and washers.
300 blunt bolts for strap-iron, $5'' \times \frac{1}{2}''$ (.127 m. × .0127 m.).
$\frac{3}{4}$ barrel Portland Cement.

Turnouts and Crossings.

To lay one typical crossing, as shown in Plate XX, consisting of 218 lineal feet (66.5 m.) of track, with five frogs, from switches and outside guard timbers, with inside steel guard rails, costs as follows:

Hoisting, adzing and clipping cross-ties..........	$18 25
Laying rails, frogs and switches..................	40 00
" guard timbers..........................	12 50
" steel guard rails........................	4 25
Total..................................	$75 00
Amount paid contractor as per prices bid............	139 30
Profit..................................	$64 30

or $85\frac{7}{10}$ per cent.

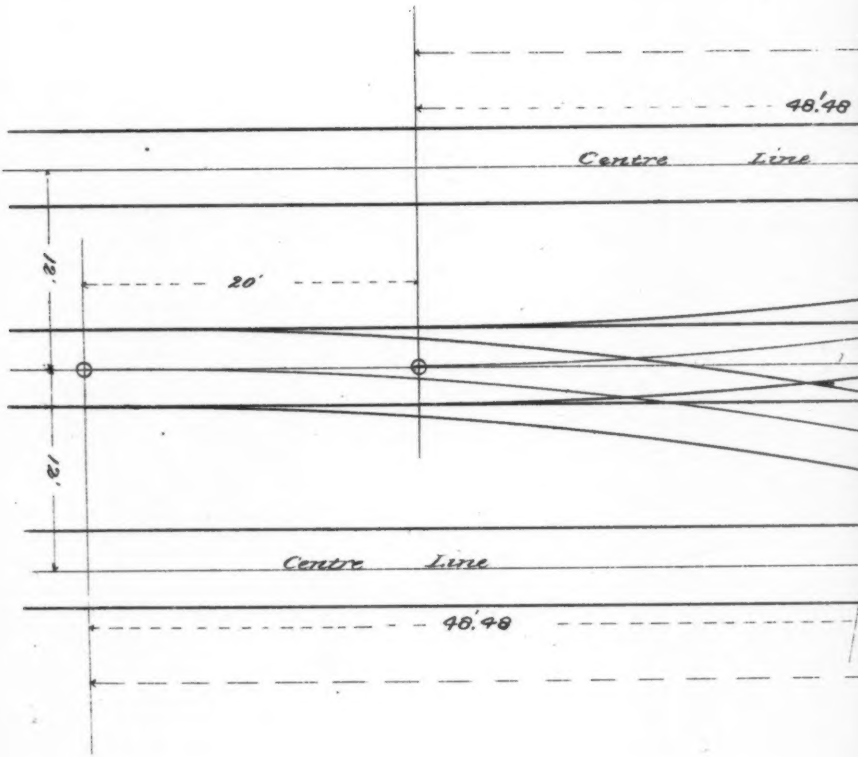
Sidewalks Three Planks Wide.

Cost to contractor per 1 000 lineal feet (304.8 m.)..	$15 00
Amount paid contractor as per prices bid.........	25 00
Profit..................................	$10 00

or $66\frac{2}{3}$ per cent.

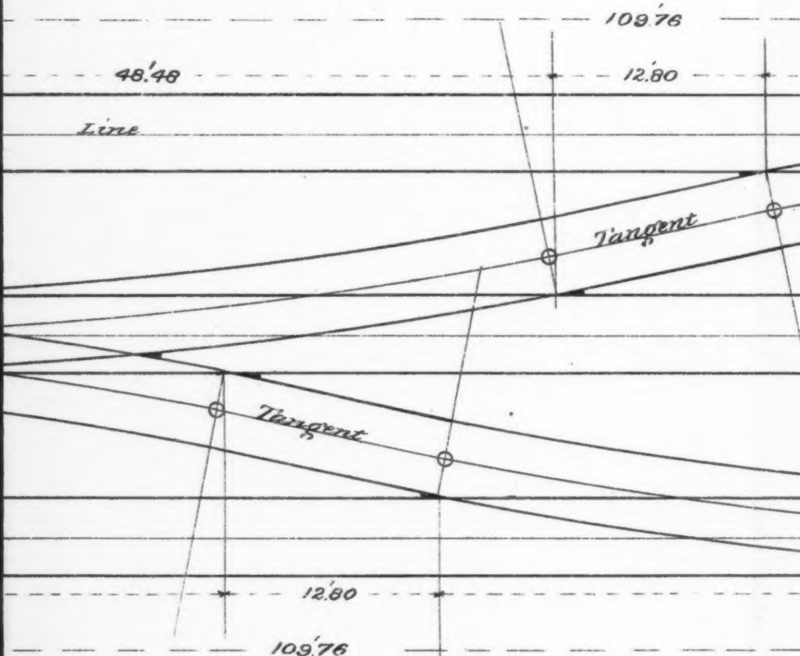
Painting.

In mixing paints for iron surfaces, it is of the first importance that the best materials only be used. Linseed oil is the best medium, when free from admixture with turpentine, the latter being a volatile oil, cannot be used with advantage on a non-absorbent surface. Linseed oil, however, is peculiarly well adapted for this purpose. It does not evaporate in any perceptible degree. The large percentage of linolein which it contains combines with the oxygen of the atmosphere, and makes the surface of the paint solid and of a resinous appearance, possessing toughness and elasticity. Linseed oil does not crack or blister, by reason of the expansion and contraction of the iron with variation of temperature. Another important characteristic is its expansion while drying, which peculiarly


48.'48
Centre Line
12'
20'
12'
Centre Line
48.'48

Typical Crossing
used on
2nd Ave. Line
M.R.R.

Scale 1in = 10ft.
" 1mm. = 120 mm.



Gauge on tangent track = 4' 8½"
" curved 4' 9"
Frog Angle = 11° 4' 30"
Middle Frog Angle 14° 12' 0"
Radius = 250'
Point of switch 6" ahead of point of cur

PLATE XX
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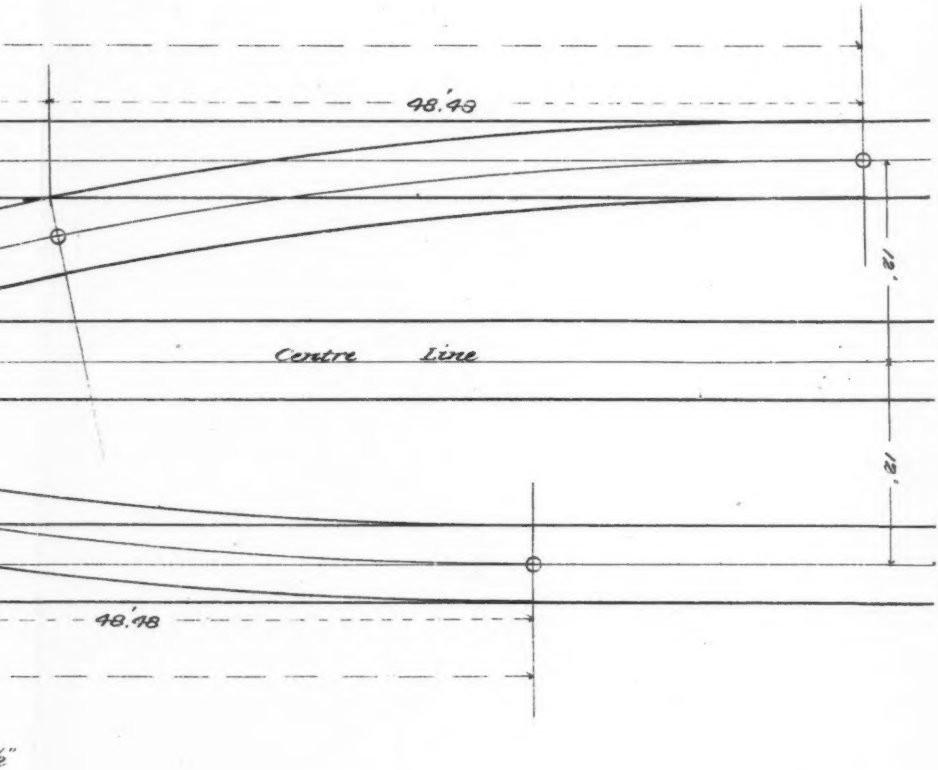


of curve.

adapts it to iron surfaces. The Metropolitan Elevated Railway Company spent no small amount of money in experimenting with various classes, kinds and colors of paint, before arriving at a final conclusion; their labors at last culminated in the selection of a metallic paint for the first coat, and a white lead paint for the second and last coat, both paints to be mixed with the best boiled linseed oil, and enough turpentine to make the paint cover well and facilitate its drying. The following formulæ were therefore adopted and adhered to:

(1.) METALLIC PAINT.

9 parts of boiled linseed oil.
1 part of turpentine.
7½ pounds of "metallic" (ground iron ore).

Made one mixed gallon (3.79 l) of paint, and covered about 256 square feet (23.78 sq. m).

(2.) WHITE LEAD PAINT, OLIVE COLOR.

325	pounds	(147.42 kg.)	white lead.
175	"	(79.38 ")	white lime.
75	"	(34.02 ")	French ochre.
3	"	(1.361 ")	Prussian blue.
1	pound	(.454 ")	burnt umber.
21	galls.	(79.50 l.)	boiled linseed oil.
1½	"	(5.678 l.)	turpentine.
1	"	(3.79 l.)	liquid drier.

Made 751 pounds (340.64 kg.) or 18 gallons (68.13 l.) of mixed paint, and covered about 512 square feet (47.56 sq. m.).

The iron work was first thoroughly cleaned, rough spots scraped, and such spots, together with all new rivets or other raw details, were "touched up" with one coat of metallic paint.

All the seams in top and bottom chords, gaps at brace joints, and all other cracks or seams needing it, were filled solid with linseed oil putty. The top surface of the top chords was then given one coat of metallic paint. The strap-iron, clips, fish-plates and under side of cross-ties, steel rails, and guard timbers, were painted one coat of same before adjustment. All cracks, joints, sun checks, &c., in the timber, were filled with putty prior to painting with metallic paint and again before using the white lead color.

The track superstructure received two coats of metallic paint. All track work being finished and the whole structure complete, the final

covering of white lead paint was laid on with great care, under close inspection, and has proved, so far, to be the desideratum for iron surface painting. The actual cost of painting the structure, including the station girders, but not their buildings, with all the details of scraping, cleaning, puttying, together with all materials and labor was $1 50 per lineal foot.

The labor account was $36\frac{8}{10}$ per cent. of the total cost.

While the material account was $63\frac{2}{10}$ per cent. of the total cost.

Appendix.

Form of estimate sheet of special street pier, built January, 1881, showing the exact construction, character of material in excavation, quantities, contract prices paid, and the total cost of the pier as ready for the column at connection with Harlem River Bridge, north end of Eighth avenue. (See Plate XXI.)

Nature of bottom—silt.

Bottom stones set—2, (7′×3′×6″).

Bolts—4-10′ 4″ = 472 lbs.

Washers—4 = 108 lbs.

Base set—1 = 3200 lbs.

Refilling (62.59+73.11) - (13.49+16.92+22.67) = 82.62 Cu. yds.

Excavation:

Earth—13′×13′×10′ = 62.59 cu. yds.

Loose rock—13′×13′×11′.68 = 73.11 cu. yds.

Pitwork—13′×13′×12′.93 = 80.93 cu. yds.

Piling—16 - 50′ = 800 lin. ft.

Capping—4 - 9′×6″×12″, 18 - 9′×6″×6″ = 702 ft., B. M.

Concrete—12′×12′×4′.25=83.46 cu. ft. Grillage = 19.58 cu. yds.

Brickwork:

Pier—4′ 6″ squ.×8′ 0″ sq.×9′.083 — 21 cu. ft. (bottom stones) = 12.71 cu. yds.

Sub-pier—8′×8′×7′.14 = 16.92 cu. yds.

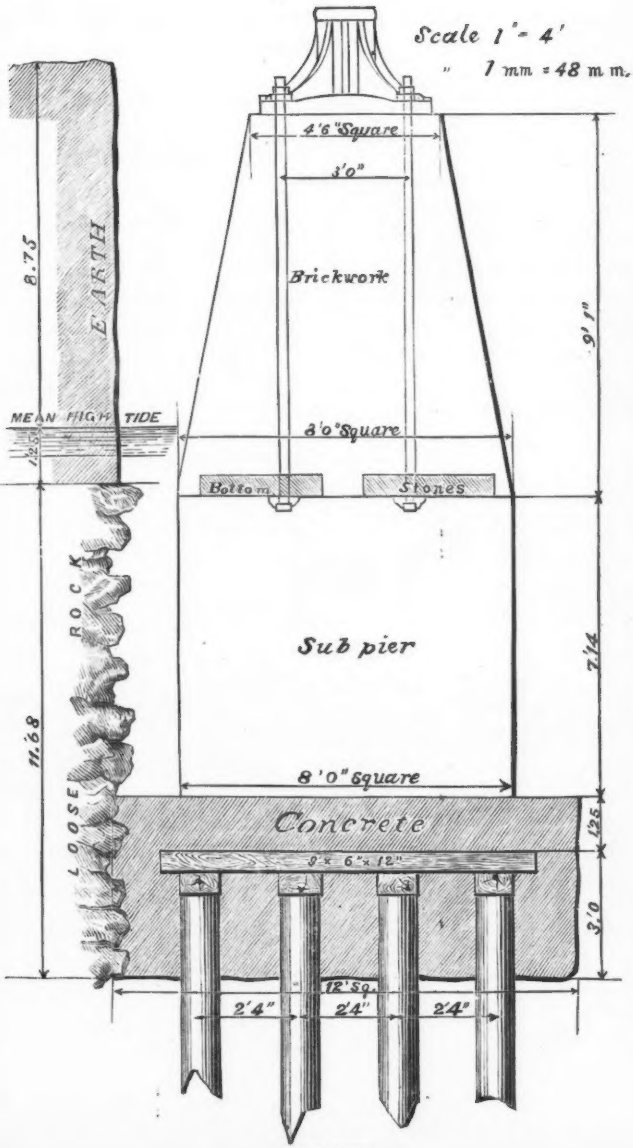
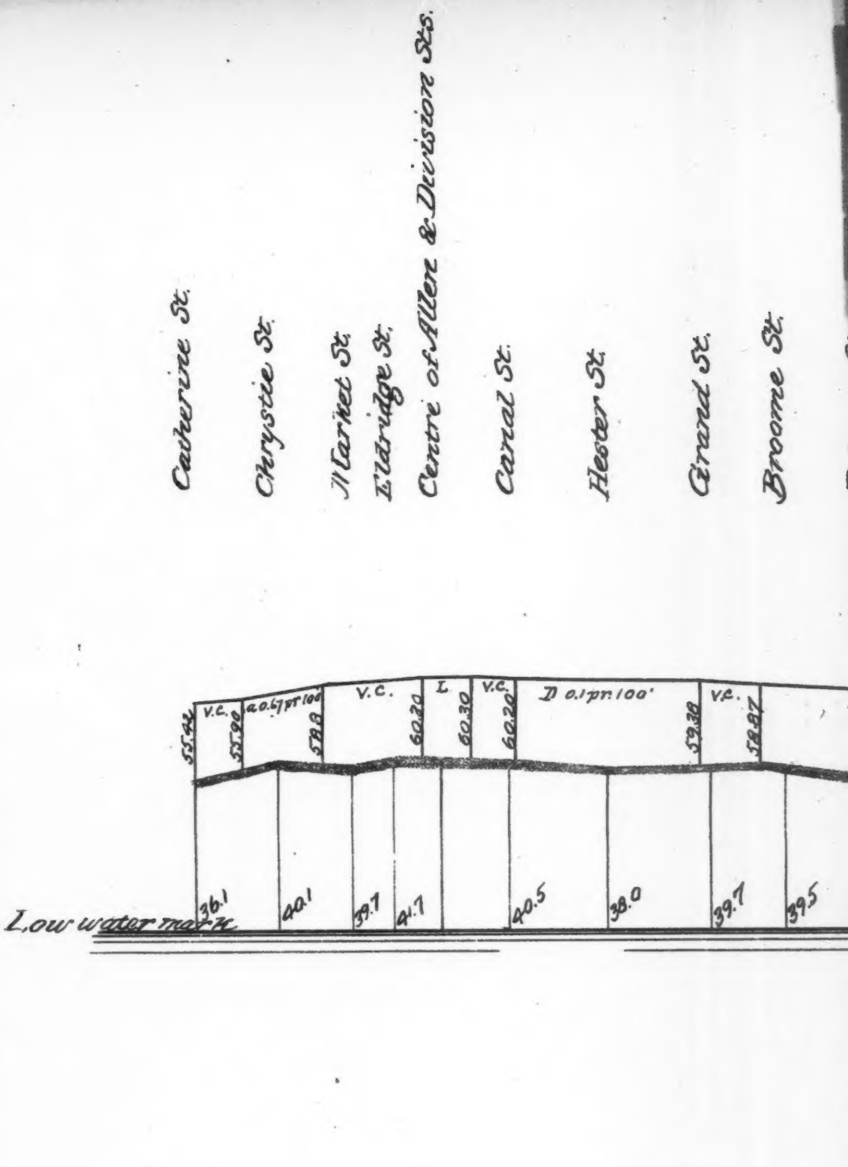

Scale 1" = 4'
" 1 mm = 48 mm.
4'6" Square
3'0"
Brickwork
9'1"
EARTH
8.75
MEAN HIGH TIDE
1.25
8'0" Square
Bottom Stones
Sub pier
7.14
LOOSE ROCK
11.68
8'0" Square
Concrete
1.25
9 × 6" × 12"
3.0
12' Sq.
2'4"
2'4"
2'4"

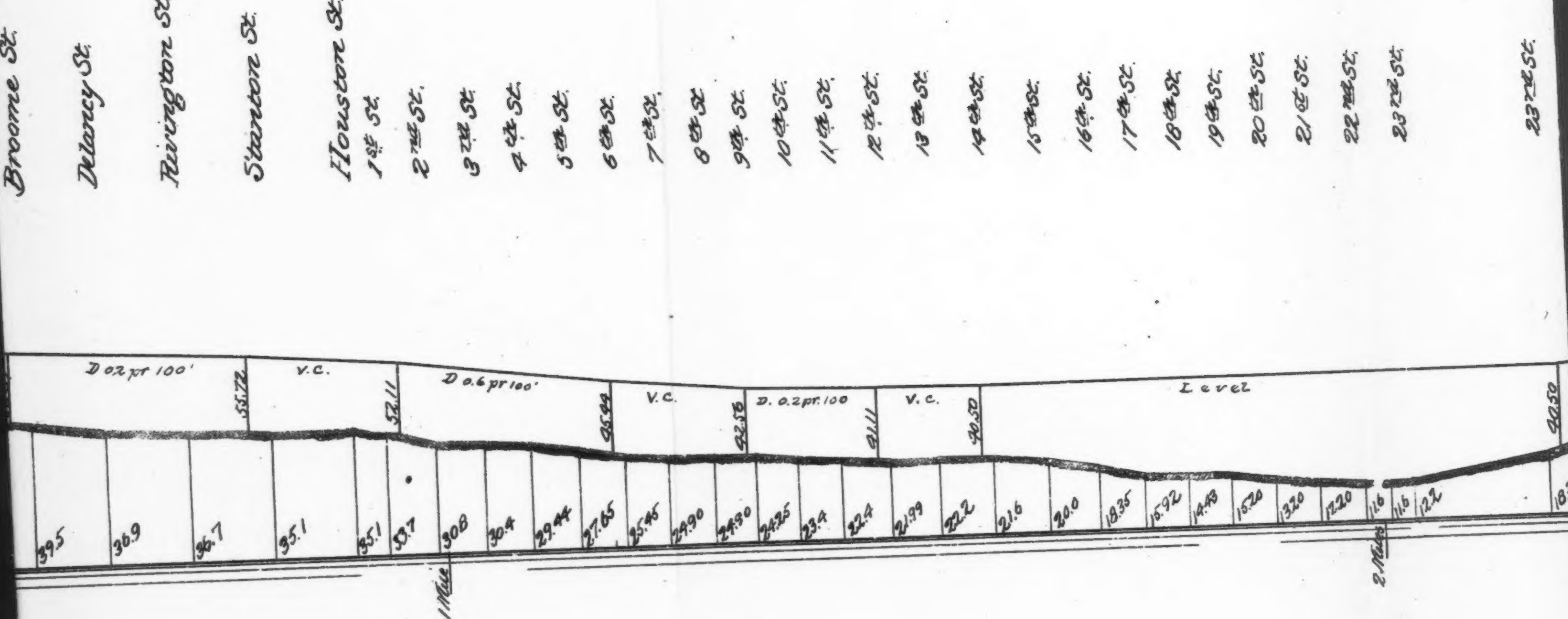
PLATE XXI.

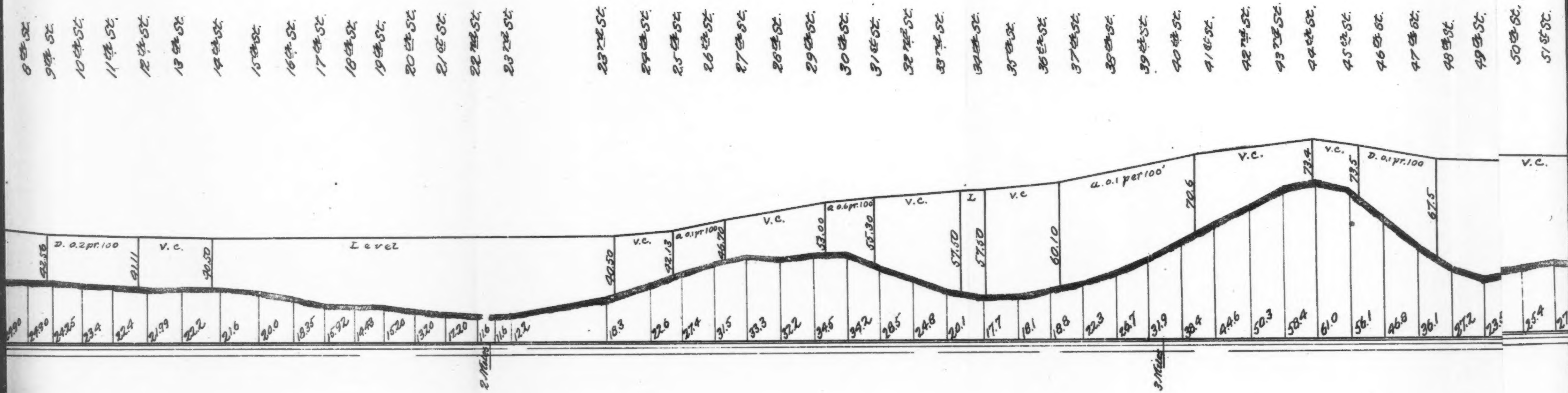
ESTIMATE OF WORK DONE AND MATERIALS FURNISHED BY M. TIERNEY, CONTRACTOR.

QUANTITIES.	ITEMS.	AMOUNT.	REMARKS.
62.59 cu. yds...	Earth Excavation @ $1 50....	$93 89	The iron and bottom stones were furnished by the company.
73.11 " " ...	Loose Rock Excavation @ $3 00	219 33	
80.93 " " ...	Pitwork @ $2 00..............	161 86	
800.—lin. ft....	Piling @ $0 30...............	240 00	
702.—ft. B. M..	Capping @ $35 00 pr. M......	24 57	
19.58 cu. yds...	Concrete @ $5 00.............	97 90	
29.63 " ...	Brickwork @ $9 00...........	266 67	
2..............	Bottom Stones @ $2 00......	4 00	
1..............	Base set.....................	4 00	
82.62 cu. yds...	Refilling @ $0 10............	8 26	
	Total................	$1 120 48	

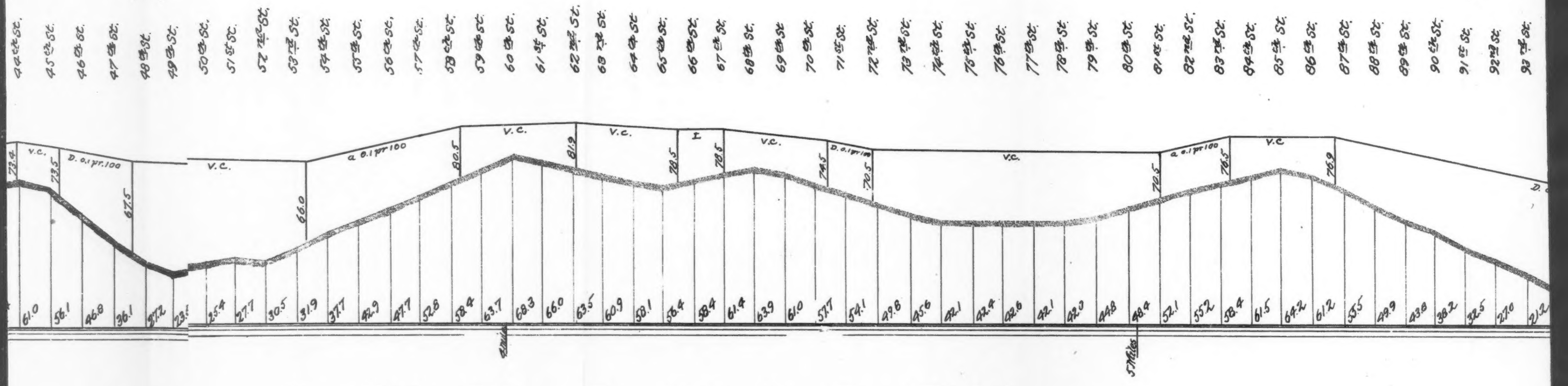


Catherine St.
Chrystie St.
Market St.
Eldridge St.
Centre of Allen & Division Sts.
Canal St.
Hester St.
Grand St.
Broome St.
55.42
V.C.
55.90
a.0.67 pr 100
58.8
V.C.
60.20
L
60.30
V.C.
60.20
D 0.1 pr. 100'
59.38
V.C.
58.87
36.1
40.1
39.7
41.7
40.5
38.0
39.7
39.5
Low water mark


Broome St.
Delancy St.
Rivington St.
Stanton St.
Houston St.
1st St.
2nd St.
3rd St.
4th St.
5th St.
6th St.
7th St.
8th St.
9th St.
10th St.
11th St.
12th St.
13th St.
14th St.
15th St.
16th St.
17th St.
18th St.
19th St.
20th St.
21st St.
22nd St.
23rd St.
23rd St.
D 0.2 pr 100'
55.72
V.C.
52.11
D 0.6 pr 100'
35.99
V.C.
32.58
D. 0.2 pr. 100
31.11
V.C.
30.50
Level
30.50
39.5
36.9
36.7
35.1
35.1
33.7
30.8
30.4
29.44
27.65
25.46
24.90
24.30
24.25
23.4
22.4
21.99
22.2
21.6
20.0
18.35
15.92
14.43
15.20
13.20
12.20
11.6
11.6
12.2
18.3
1 Mile
2 Miles



D. 0.2 pr. 100
V.C.
Level
a. 0.1 per 100'
D. 0.1 pr. 100
2 Miles
3 Miles



PROFILE

2nd Avenue Line, Manhattan Railway Co.

Scale { Horizontal
Vertical

PLATE XXII.
TRANS. AM. SOC. CIV. ENGR'S
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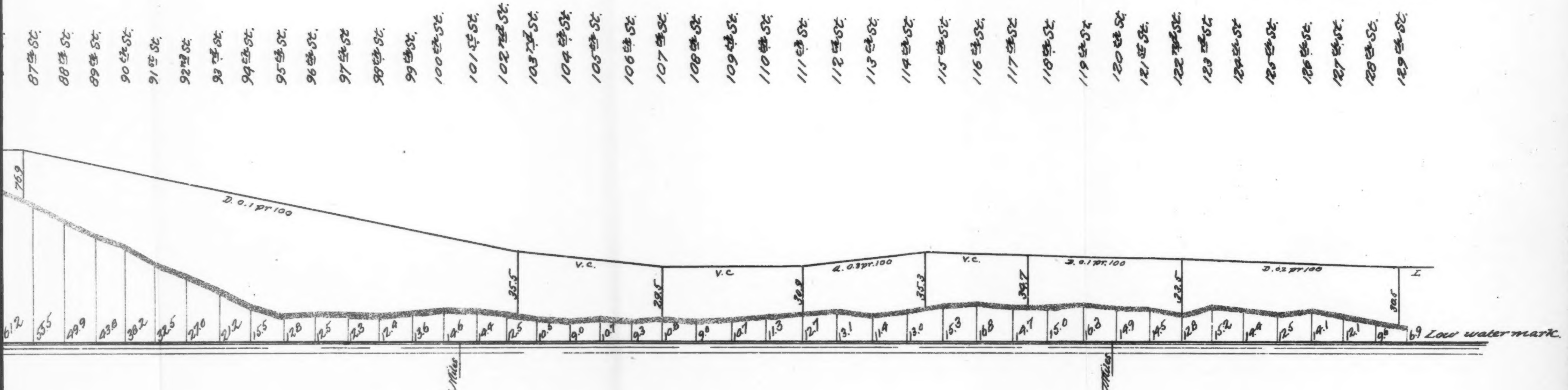
HALL ON ELEVATED RAILROAD



AMERICAN SOCIETY OF CIVIL ENGINEERS

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CCXIX.

(Vol X.—April, 1881.)

EXPONENT OF THE PRINCIPLE OF MOMENTS.

By W. S. Auchincloss, Member of the Society.

Read March 2d, 1881.

The processes of mathematics and mechanics have, in many respects, mutual relations, like those existing between force, light and heat. Many of the formulae of mathematics find direct expression in mechanical devices, so that the mind naturally glides from one to the other. The greater the attention paid to this characteristic, the clearer will the principles of mathematics appear, and the more marked the rapidity with which solutions can be effected.

The averaging machine, illustrated herewith (see Plate XXIII), is an exponent of the "Principle of Moments." In the development of this machine, the first effort was to determine a simple rule for computing average dates. For this purpose the "Principle of Moments" was found applicable, and a rule constructed thereby. The next step was to give mechanical expression to the rule. This has been secured by the device represented in the adjoining cut.

The machine consists of a scale and a series of weights. The scale, when not ladened, maintains its equilibrium *irrespective* of the position of the scale pan. The arm of the beam has 31 notches, representing the days of the month, and the scale pan is hung on a small saddle, capable of being moved from end to end of the beam. A carrier bar is fastened directly over the scale arm, and upon it a counterweight slides freely. This counterpoise exactly equals in weight the scale pan with its saddle. Two delicate watch chains are attached to opposite sides of the counterweight. They pass around little carrier wheels secured to the extremities of the arm, and are fastened to opposite sides of the saddle. In this way the saddle and the counterweight become, as it were, links in an endless chain, so that the counterpoise responds instantly to the slightest motion of the scale pan, and maintains the equilibrium of the system, for all positions of the same. By this device the weight of the pan is no longer a factor in the problem, but in effect the pan is rendered imponderable. The two balls shown at the extremity of the scale arm are used simply for purposes of adjustment as customary on all scales. Directly over the fulcrum are the usual index pointers. The platform of the scale has 31 transverse groves. These are arranged equidistant, and are capable of receiving the weights. The platform is hinged to the opposite arm of the scale, and is surrounded by a metallic fence, that is shaped like a spout on the far side. This spout serves to guide the weights in their descent to the separator, after the solution of any problem. Each groove is properly numbered from 1 to 31, inclusive, to correspond with the numbers on the scale arm. The various problems of "average date" are determined by the use of five varieties of weights shown in the cut, under letters *A*, *B*, *C*, *D* and *E*. The *A* and *C* balls are made of lead. The *B* and *D* cylindrical bodies are made from wrought-iron rods. All of these weights are nickel-plated to prevent soiling the hands. Each ball, *C*, is equal in weight to 10 balls of *A*, while each weight, *E*, is equal to 10 of *C*. In this way *A*, *C* and *E* may represent; units, tens and hundreds; or 10, 100, 1 000; or 100, 1 000, 10 000; and so on; expressing as the occasions requires, whole numbers or decimals. The weights *B* and *D*, are used simply for the purpose of economizing time. One of *B* equals 5 of *A*, and one of *D* equals 5 of *C*. The use of these intermediates saves tedious counting of the balls, *A* and *C*, and their characteristic form prevents possibility of mistakes.

It remains only to describe the separator before explaining the mode of using the machine. The separator is located directly under the platform. The balls, when dumped by the latter, are received on an inclined plane, which is covered with rubber to deaden the sound. This incline causes the balls to roll to the front part of the machine, where they fall upon a wire screen. This is of suitable size to allow *A* balls to drop into their own compartment, but retains the *C* balls, thus effecting a perfect separation. The weights *B* and *D*, should be lifted from the platform and lodged in cups on the right and left of the separator.

When it is required to determine the average date of a number of purchases made during any month, it is only necessary to place weights representing the amounts purchased in the grooves representing the days, fill the scale pan with exactly the same amount of weight as placed on the platform, then move the pan along the scale arm until the weights in the pan *exactly balance* those on the platform. The reading of the scale arm will give the "average date" of the purchases to which 30, 60 or 90 days must be added according to terms of sale.

The woodcut shows but one form of averaging machine, but as occasion requires, the number of grooves can be greatly increased and the machine adapted to various requirements.

The machine can be used for solving a great variety of problems, by varying the grooves, notches and weights. If, for instance, a vertical line passing through the fulcrum is made to exactly divide the system of grooves and notches, so that all are equidistant, and no blank spaces intervene between the line and the No. 1 groove, or the No. 1 notch; then the machine will solve a vast variety of problems of direct and inverse proportion; as for instance, the diameters and speeds of pulleys; the diameters, circumferences and areas of circles, of ellipses, and so forth. With speed problems it is only necessary to let the grooves of the platform represent the diameters of the large pulleys, and the notches on the arm the diameters of the small pulleys; whereupon the speed of the large pulley, will be represented by whatever weight placed in the groove will balance the weight in the pan, which latter represents the speed of the small pulley. It is evident that if any three quantities are given, the fourth can at once be determined.

Again, if the distance of any groove from the fulcrum be taken as unity, and the scale pan located at a distance in the opposite direction equal to 3.14159+, then any weight in the pan that will balance a given

weight in the groove will represent the diameter of a circle ; while the weight in the groove will represent the circumference of the same circle.

By using weights of different specific gravities in the pan and on the platform, or else by using specially graduated scales, problems in square root can be determined with like facility.

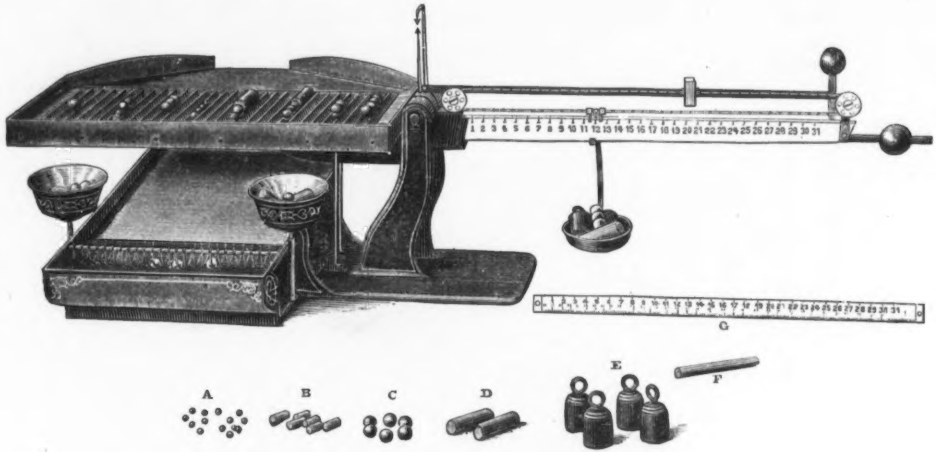
For every day services these capabilities are of little moment, as compared with the process from which the machine derives its name. They are, however, of interest to the student, for they clearly illustrate the intimacy of the relation existing between the processes of mathematics and mechanics.

In the matter of averaging commercial accounts, the machine leaves the workings of the mind far in the distance, for by its aid one can solve 100 accounts per hour without fatigue, or uncertainty as to results. The machine has an additional advantage, for it can be successfully operated by those who have but little skill with figures.

The writer ventures the opinion that *for every formula or rule in mathematics* (possibly excepting higher mathematics) *a suitable mechanism can be devised which will perfectly illustrate and express the same.*

It is not claimed that in every class of problem the extreme precision of a mathematical solution can be attained, for as a matter of expense, it may not be expedient to seek a perfection of adjustment that will insure such results. This is illustrated in the case of the averaging machine. It would be possible to adjust it with the perfection found alone in an assayer's balance, so that the wing of a fly, or even the scratch of a pencil would affect the equilibrium, but the outlay would be entirely unnecessary ; and what would it signify ? The machine might indicate a certain payment should be made at 3.30 A. M., when in practice no one could be found, at that early hour, either to pay or to receive the money.

It may be objected, that the machine does not, in the case of heavy transactions, take notice of amounts less than ten dollars. This is true, but there is no reason why weights should not be made to represent any desired amount. It is equally true that the influence of such weights on the final result would scarcely be appreciable ; and, at most, would only modify it a few hours in time. Practice will, in all cases, indicate what degree of mechanical nicety should be attained in the construction of any machine.



AVERAGING MACHINE.

PLATE XXIII.

PATENTED DEC. 21, 1880.